In a nutshell: Approximating derivatives using interpolating polynomials

In this topic, we assume that the x or t values are equally spaced. We will assume that equal spacing equals h. The derivative is being approximated at x_k or t_k, as applicable.

Derivative formulas

Centred divided-difference

$$\frac{f(x_{k+1}) - f(x_{k-1})}{2h} = \frac{f(x_k + h) - f(x_k - h)}{2h}$$

Backward divided-difference

$$\frac{y(t_k) - y(t_{k-1})}{h} = \frac{y(t_k) - y(t_k - h)}{h}$$

$$\frac{3y(t_k) - 4y(t_{k-1}) + y(t_{k-2})}{2h} = \frac{3y(t_k) - 4y(t_k - h) + y(t_k - 2h)}{2h}$$

Second derivative formulas

Centred divided-difference

$$\frac{f(x_{k+1}) - 2f(x_k) + f(x_{k-1})}{h^2} = \frac{f(x_k + h) - 2f(x_k) + f(x_k - h)}{h^2}$$

Backward divided-difference

$$\frac{y(t_k) - 2y(t_{k-1}) + y(t_{k-2})}{h^2} = \frac{y(t_k) - 2y(t_k - h) + y(t_k - 2h)}{h^2}$$

$$\frac{2y(t_k) - 5y(t_{k-1}) + 4y(t_{k-2}) - y(t_{k-3})}{h^2} = \frac{2y(t_k) - 5y(t_k - h) + 4y(t_k - 2h) - y(t_k - 3h)}{h^2}$$